ANNAMALAI UNIVERSITY

(Accredited with 'A+" Grade by NAAC)

Directorate of Distance Education

S018 M.Sc. Mathematics - Fourth Semester - 2022-2023

Instructions for submission of Assignment for Fourth Semester:

1. Following the introduction of semester pattern, it becomes mandatory for candidates to submit assignments for all the courses of the Programme. Assignment topics for all the courses are intimated to candidates through website (www.audde.in).
2. Assignments for each course carry 25 marks. Candidates should submit assignments for each Course / subject separately. (5 Questions x 5 Marks = 25 marks).
3. The answers for each Course / subject should not exceed 20 pages.
4. Assignments should be in the own handwritten of the student concerned and not type-written or printed or photocopied.
5. Assignments should be written on A4 Paper on one side only. Write your Enrollment number on the top right corner of all the pages.
6. Add a template page and provide details regarding your Name, Enrollment number, Programme name, Course code and Title. Assignments without duly filled in template will not be accepted.
7. Assignments should be handwritten only. Typed or Printed or Photocopied assignments will not be accepted.
8. All assignments (with enrolment number marked on the top right hand corner on all pages) should be put in an envelope with superscription ***"M.Sc. Mathematics – Assignments – IV Semester"***
9. **Send all assignments of the Fourth semester in one envelope** and send to ***The Director, Directorate of Distance Education, Annamalai University, Annamalai Nagar – 608 002, Tamil Nadu*** by Registered Post

Dates to Remember

Last date for submission of Assignments of Fourth Semester	: 15.04.2023
Last Date for submission with late fee ` 300/-	:29.04.2023

Dr. R. SINGARAVEL
Director

Assignment Question

018E2410 Complex Analysis- II

1. (a) Prove that $\int_0^{2\pi} \frac{d\theta}{1+a\sin\theta} = \frac{2\pi}{\sqrt{1-a^2}} (-1<a<1).$

   (b) State and prove Schwarz's theorem.

   (C) State Reflection principle and Hadamard's theorem.

   (d) Represent sin π z in the form of canonical product.

2. (a) State and prove Weierstrass theorem and Laurent's theorem

   (b) Find the poles and residues at their poles of the following:

   (i) $\frac{z^2}{z^2+a^2}$, (ii) $\frac{1}{(z^2-1)^2}$

3. (a) State and prove Jensen's formula and Poisson-Jensen formula.

   (b) Prove that the infinite product $\pi_1^{\infty}\left(1+\frac{z}{n}\right)e^{\frac{-z}{n}}$ converges uniformly

   and absolutely on every compact subset.

4. (a) State and prove Arzela's theorem.

   (b) Prove that, the zeros $a_1, a_2, ..., a_n$ and poles $b_1, b_2, ..., b_n$ of an elliptic function satisfy $a_1 + a_2 + ... + a_n \equiv b_1 + b_2 + ... b_n$ (mod M).

   (c) (i) Define normal family of functions.

   (ii) Prove that $P(z) - P(u) = -\frac{\sigma(z-u)\sigma(z+u)}{\sigma(z)^2\sigma(u)^2}$.

5. (a) Prove that a family F is normal if and only if its closure $\overline{\text{F}}$ with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty}\delta_k(f,g)2^{-k}$ is compact.

   (b) There exists a basis (ω_1, ω_2) such that the ratio $\gamma = \omega_2 / \omega_1$ satisfies the following conditions:

   (i) Im $\gamma > 0$,

   (ii) $-\frac{1}{2} < \text{Re}\,\gamma < \frac{1}{2}$,

   (iii) $|\gamma| > 1$,

   and (iv) Re $\gamma > 0, if \mid \gamma \mid = 1$.

   The ratio γ is uniquely defined by these conditions.

1.

018E2420 Function Analysis

1. (i) Define normed linear space and Banach space

   (ii) State and prove Minkowski's inequality

   (iii) Prove the Theorems: Hahn – Banach and Open- Mapping

2. (a) State a d prove (i) Schwartz inequality (ii) Bessel's inequality and Triangular inequality.

   (b) If it is a complex Banach s[ace whose norm obeys

   $\|x+y\|^2 + \|x-y\|^2 = 2\|x\|^2 + 2\|y\|^2$ and

   4(x,y) = $\|x+y\|^2 - \|x-y\|^2$ + $i\|(x+iy)\|^2$- $i\|(x-iy)\|^2$

   Prove that B is a Hilbert Space.

   (c) If p is a projection on H which range M and null space N, prove that M⊥N ⇔ p is self adjoint, and in this case $N = M^{\perp}$.

3. (a) If T is normal, then prove that the eigen spaces of T are pairwise orthogonal

   (b) If B=$\{e_i\}$ is a basis for H, prove that the matrix relative to B, is an isomorphism of the algebra β(H) onto the Matrix algebra A_n

4. (a) Show that the mapping $x \to x^{-1}$ of G into G is continuous and is Therefore a homeomorphism of G onto itself.

   (b) Prove that $\| f * g \| = \| f \| \| g \|$.

5. (a) *If* G is open in a Banach Algebra A then prove that S is closed.

   (b) If I is a proper closed two-sided ideal in A, Prove that the quotient Algebra A/I is a Banach Algebra.

018E2430 Mathematical Statistics

1. (a) State and Prove Borel-Canteli Lemma

   (b) If all the correlation coefficients of zero orders in a set of p-variates are equal to ρ, show that

   i) Every partial correlation of s-th order is $\frac{\rho}{(1+s\rho)}$ and

   ii) The coefficient of multiple correlations R of a variate with the other (p-1) variates is given by

   $1-R^2 = (1-\rho)\left[\frac{1+(p-1)\rho}{1+(p-2)\rho}\right].$

2. (a) State Kolmogorov Inequality and prove it.

   (b) Define random sampling and Analysis of Variance.

   (c) If T_n is a sequence of estimates such that $E(T_n) \to \theta$ and $Var(T_n) \to 0$ as $n \to \infty$, prove that T_n is consistent for θ.

3. (a) Prove that, $(n-1)\, S^2/\sigma^2$ is $\chi^2(n-1)$.

   (b) If $(X_1,Y_1), (X_2,Y_2),\ldots, (X_n,Y_n)$, $n \geq 2$ be a sample from a bivariate normal population with parameters $EX = \mu_1, EY = \mu_2$, $var(X)= \sigma^2_1$ $var(Y)= \sigma^2_2$ and $cov(X,Y)=0$. In other words, let $X_1,X_2,\ldots, X_n$ be iid $\mathcal{N}(\mu_1, \sigma^2_1)$ random variables and $Y_1,Y_2,\ldots,,Y_n$ be iid $N(\mu_2, \sigma^2_2)$ random Variables, and suppose that X's and Y's are independent. Then prove that the pdf of R is given by $f_1(r)=$

   $$\begin{cases} \frac{\Gamma(n-1)/2}{\Gamma 1/2 \Gamma n-2/2}(1-r^2)^{n-4/2} & -1 \leq r \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

4. The life times (in hours) of samples from three different brands of batteries. We recorded with the following results.

Brand

X	Y	Z
40	60	60
30	40	50
50	55	70
50	65	65
50	-	75
-	-	40

   Test the hypothesis that the three brands have different average Life times.

5. (a) State and Prove Neymann-Pearson Lemma

(b) Find a UMP size α test of H_0: $\theta \leq \theta_0$ against H_1: $\theta > \theta_0$ based on a sample of n observations for the following families of pdf's $f_\theta(x)$; $\theta \ni \theta \subseteq R$,

$$f_\theta(x) = \frac{1}{\sqrt{2\pi}} e^{-(x-\theta)^2/2}; -\infty < x < \infty; -\infty < \theta < \infty;$$

018E2440 Optimization Techniques

1. (a) Use Simplex method to solve

$$\max z = 4x_1 + 10x_2$$

Subject to the constraints

$$2x_1 + x_2 \leq 50$$
$$2x_1 + 5x_2 \leq 100$$
$$2x_1 + 3x_2 \leq 90$$
$$x_1 \geq 0 \text{and} x_2 \geq 0$$

(b) Show that, the set of all feasible solutions to the linear programming problem in convex set

2. (a) Use Big-M method to solve

$$\max z = x_1 + 2x_2 + 3x_3 - x_4$$

Subject to the constraints

$$x_1 + 2x_2 + 3x_3 = 15$$
$$2x_1 + x_2 + 5x_3 = 20$$
$$x_1 + 2x_2 + x_3 + x_4 = 10$$
$$x_i \geq 0, \quad i = 1 \text{ to } 4$$

(b) State and prove Gomory's integer cutting Algorithm.

3. (a) Use Dual Simplex method to solve

$$\min z = 3x_1 + x_2$$

Subject to the constraints

$$x_1 + x_2 \geq 1$$
$$2x_1 + 3x_2 \geq 2$$
$$x_1, x_2 \geq 0$$

(b) State and prove Duality theorem

4. (a) Use Revised Simplex method to solve

$$\max z = 3x_1 + 5x_2$$

Subject to the constraints

$$x_1 \leq 4$$
$$x_2 \leq 6$$
$$3x_1 + 2x_2 \leq 18$$

(b) How to find the inverse of new basis from the proceeding basis by application of the elimination formulas and give the examples.

5. (a) Solve the following Transportation problem

			Supply
16	20	12	200
14	8	18	160
26	24	16	90

Demand	180	120	150

(b) (i) Define three uses of revised simplex procedure.

(ii) State that perturbation techniques.

(iii) Define Degeneracy and Anti-cycling procedures.

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